

Introduction to Statistics — Quiz #3 (60 minutes)

May 21, 2026 (Thursday)

Section: _____ Cadet Number: _____ Name: _____ Score: _____

- All solutions must include a detailed step-by-step explanation.
- If an answer has more than four decimal places, round to the **fourth decimal place**.
- **Reference table** is provided on the last page of the exam.

1. Fill in each blank with an appropriate term chosen from the word box below, or determine whether each statement is **True (O)** or **False (X)**. [15 points]

null hypothesis	alternative hypothesis	Type I error	Type II error
test statistic	rejection region	significance level	power
normal distribution	sampling distribution	sample mean	sample variance
p-value	critical value	estimator	confidence level
random variable	central limit theorem	hypothesis testing	acceptance region

- (1) A (_____) is the default skeptical assumption currently believed to be true, and is denoted H_0 .
- (2) Hypothesis testing based on a sample can lead to two types of errors: the error of incorrectly rejecting H_0 when H_0 is true is called a (_____), and the error of failing to reject H_0 when H_A is true is called a (_____).
- (3) The probability of rejecting H_0 when the alternative hypothesis H_A is true, i.e., $P(\text{reject } H_0 \mid H_A \text{ true})$, is called the (_____).
- (4) A statistic used to conduct a hypothesis test based on a random sample is called a test statistic. (O / X)
- (5) The rejection region is the set of test-statistic values for which H_0 is not rejected at significance level α . (O / X)

2. A hat manufacturer wants to check whether the average head size of new recruits is 24 inches. To investigate, 25 recruits were randomly selected and their head sizes $X_1, X_2, \dots, X_{25}$ were measured. The manufacturer wishes to test $H_0 : \mu = 24$ vs. $H_A : \mu \neq 24$ at significance level $\alpha = 0.05$. Assume that head size follows a normal distribution with variance $\sigma^2 = 4$. The test statistic is $Z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}}$, where $\bar{X}$ is the sample mean, μ_0 is the null value, and n is the sample size. [40 points]

(1) Find the null distribution of the test statistic.

(2) Find the rejection region at significance level $\alpha = 0.05$.

(3) The sample mean of the 25 selected recruits is $\bar{x} = 25$ inches. Using the rejection region in (2), complete the hypothesis test. State the conclusion in the context of the data.

3. A company produces steel components whose average breaking strength is known to be $\mu_0 = 655$ MPa. An outside laboratory claimed that the true average breaking strength is now below 655 MPa. To investigate this claim, $n = 9$ steel components are randomly selected from the production line and their breaking strengths $X_1, X_2, \dots, X_9$ are measured. The company wishes to test this claim at significance level $\alpha = 0.05$. Assume that breaking strength follows a normal distribution. [45 points]

(1) State the null and alternative hypotheses.

(2) Find the test statistic and its null distribution.

(3) The breaking strengths (MPa) of the 9 randomly selected steel components are given below. Find the observed value of the test statistic.

Component (i)	1	2	3	...	6	7	8	9
Breaking strength (x_i , MPa)	621	676	634	...	678	651	646	620

$$\ast \sum_{i=1}^9 x_i = 5832, \quad \sum_{i=1}^9 (x_i - \bar{x})^2 = 3528$$

(4) Using the result in (3), report the p -value and complete the hypothesis test. State the conclusion in the context of the data.

Reference Table Let $Z \sim N(0, 1)$.

$P(Z < 0.5) = 0.6915$	$P(Z < 1) = 0.8414$	$P(Z < 1.0607) = 0.8556$	$P(Z < 1.3416) = 0.9101$
$P(Z < 1.5) = 0.9332$	$P(Z < 2) = 0.9772$	$P(Z < 2.451) = 0.9929$	$P(Z < 3) = 0.9987$
$z_{0.005} = 2.5759$	$z_{0.01} = 2.3263$	$z_{0.025} = 1.96$	$z_{0.05} = 1.6449$
$t_{0.0085}(8) = 3$	$t_{0.0185}(8) = 2.5$	$t_{0.0403}(8) = 2$	$t_{0.1733}(8) = 1$
$t_{0.005}(8) = 3.3554$	$t_{0.01}(8) = 2.8965$	$t_{0.025}(8) = 2.3060$	$t_{0.05}(8) = 1.8595$
$t_{0.005}(9) = 3.2498$	$t_{0.01}(9) = 2.8214$	$t_{0.025}(9) = 2.2622$	$t_{0.05}(9) = 1.8331$
$t_{0.0075}(9) = 3$	$t_{0.0169}(9) = 2.5$	$t_{0.0383}(9) = 2$	$t_{0.1717}(9) = 1$