Introduction to Statistics — Quiz #3 (60 minutes)

May 21, 2026 (Thursday)

Section: ________ Cadet Number: ______________ Name: ________________ Score: _____________

- All solutions must include a detailed step-by-step explanation.
- If an answer has more than four decimal places, round to the **fourth decimal place**.
- **Reference table** is provided on the last page of the exam.

1. Fill in each blank with an appropriate term chosen from the word box below, or determine whether each statement is **True (O)** or **False (X)**. [15 points]

null hypothesis	alternative hypothesis	Type I error	Type II error
test statistic	rejection region	significance level	power
normal distribution	sampling distribution	sample mean	sample variance
p-value	critical value	estimator	confidence level
random variable	central limit theorem	hypothesis testing	acceptance region

(1) A (____________________) is the default skeptical assumption currently believed to be true, and is denoted H_0.

(2) Hypothesis testing based on a sample can lead to two types of errors: the error of incorrectly rejecting H_0 when H_0 is true is called a (____________________), and the error of failing to reject H_0 when H_A is true is called a (____________________).

(3) The probability of rejecting H_0 when the alternative hypothesis H_A is true, i.e., $P(\text{reject } H_0 \mid H_A \text{ true})$, is called the (____________________).

(4) A statistic used to conduct a hypothesis test based on a random sample is called a test statistic. (O / X)

(5) The rejection region is the set of test-statistic values for which H_0 is not rejected at significance level α. (O / X)

<u>Solution:</u> null hypothesis / Type I error / Type II error / power / O / X

2. A hat manufacturer wants to check whether the average head size of new recruits is 24 inches. To investigate, 25 recruits were randomly selected and their head sizes $X_1, X_2, \ldots, X_{25}$ were measured. The manufacturer wishes to test $H_0 : \mu = 24$ vs. $H_A : \mu \neq 24$ at significance level $\alpha = 0.05$. Assume that head size follows a normal distribution with variance $\sigma^2 = 4$. The test statistic is $Z = \dfrac{\bar{X} - \mu_0}{\sigma/\sqrt{n}}$, where $\bar{X}$ is the sample mean, μ_0 is the null value, and n is the sample size. [40 points]

(1) Find the null distribuiton of the test statistic.

<u>Solution:</u> Since $\sigma^2 = 4$ (i.e., $\sigma = 2$) is known and $n = 25$, $Z = \frac{\bar{X}-\mu_0}{\sigma/\sqrt{n}} = \frac{\bar{X}-24}{2/\sqrt{25}} = \frac{\bar{X}-24}{0.4} \sim N(0,\,1)$.

(2) Find the rejection region at significance level $\alpha = 0.05$.

<u>Solution:</u> Since this is a two-sided test, the rejection region is

$$|z| > z_{0.025} = 1.96.$$

(3) The sample mean of the 25 selected recruits is $\bar{x} = 25$ inches. Using the rejection region in (2), complete the hypothesis test. State the conclusion in the context of the data.

<u>Solution:</u> The observed test statistic is

$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{25 - 24}{0.4} = 2.5.$$

Since $|z| = 2.5 > 1.96$, we reject H_0. There is sufficient evidence at $\alpha = 0.05$ to conclude that the average head size of recruits differs from 24 inches.

3. A company produces steel components whose average breaking strength is known to be $\mu_0 = 655$ MPa. An outside laboratory claimed that the true average breaking strength is now below 655 MPa. To investigate this claim, $n = 9$ steel components are randomly selected from the production line and their breaking strengths $X_1, X_2, \ldots, X_9$ are measured. The company wishes to test this claim at significance level $\alpha = 0.05$. Assume that breaking strength follows a normal distribution. [45 points]

(1) State the null and alternative hypotheses.

Solution:

$$H_0 : \mu = 655, \qquad H_A : \mu < 655.$$

(2) Find the test statistic and its null distribution.

Solution: Since the population variance σ^2 is unknown, we use the T statistic. When H_0 is true,

$$T = \frac{\bar{X} - \mu_0}{S/\sqrt{n}} = \frac{\bar{X} - 655}{S/\sqrt{9}} = \frac{\bar{X} - 655}{S/3} \sim t(8).$$

(3) The breaking strengths (MPa) of the 9 randomly selected steel components are given below. Find the observed value of the test statistic.

Component (i)	1	2	3	$\cdots$	6	7	8	9
Breaking strength $(x_i,\ \mathrm{MPa})$	621	676	634	$\cdots$	678	651	646	620

※ $\sum_{i=1}^{9} x_i = 5832, \quad \sum_{i=1}^{9} (x_i - \bar{x})^2 = 3528$

Solution:

$$\bar{x} = \frac{1}{9}\sum_{i=1}^{9} x_i = \frac{5832}{9} = 648, \qquad s = \sqrt{\frac{1}{9-1}\sum_{i=1}^{9}(x_i - \bar{x})^2} = \sqrt{\frac{3528}{8}} = 21.$$

The observed test statistic is

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{648 - 655}{21/\sqrt{9}} = \frac{-7}{7} = \boxed{-1}.$$

(4) Using the result in (3), report the p-value and complete the hypothesis test. State the conclusion in the context of the data.

Solution: Since $H_A : \mu < 655$ is a one-sided (left-tailed) test,

$$p\text{-value} = P(T < t_0^*) = P(T < -1) = \boxed{0.1733}.$$

Since $p\text{-value} = 0.1733 > \alpha = 0.05$, we fail to reject H_0. There is insufficient evidence at $\alpha = 0.05$ to support the claim that the average breaking strength of the steel components is less than 655 MPa.

Reference Table Let $Z \sim N(0, 1)$.

$P(Z < 0.5) = 0.6915$	$P(Z < 1) = 0.8414$	$P(Z < 1.0607) = 0.8556$	$P(Z < 1.3416) = 0.9101$
$P(Z < 1.5) = 0.9332$	$P(Z < 2) = 0.9772$	$P(Z < 2.451) = 0.9929$	$P(Z < 3) = 0.9987$
$z_{0.005} = 2.5759$	$z_{0.01} = 2.3263$	$z_{0.025} = 1.96$	$z_{0.05} = 1.6449$
$t_{0.0085}(8) = 3$	$t_{0.0185}(8) = 2.5$	$t_{0.0403}(8) = 2$	$t_{0.1733}(8) = 1$
$t_{0.005}(8) = 3.3554$	$t_{0.01}(8) = 2.8965$	$t_{0.025}(8) = 2.3060$	$t_{0.05}(8) = 1.8595$
$t_{0.005}(9) = 3.2498$	$t_{0.01}(9) = 2.8214$	$t_{0.025}(9) = 2.2622$	$t_{0.05}(9) = 1.8331$
$t_{0.0075}(9) = 3$	$t_{0.0169}(9) = 2.5$	$t_{0.0383}(9) = 2$	$t_{0.1717}(9) = 1$