

Introduction to Statistics - Quiz #4(60 minutes)

June 4, 2026 (Thursday)

Section(교반): _____ Cadet Number(교번): _____ Name(성명): _____ Score: _____

- All solutions must include a detailed step-by-step explanation.
- If an answer has more than four decimal places, round to the **fourth decimal place**.
- **Reference table** is provided on the last page of the exam.

1. The driving distance per full charge (in km) of an electric vehicle is known to follow a normal distribution: with a new battery (A), mean μ_A and variance $\sigma_A^2 = 18$; with the existing battery (B), mean μ_B and variance $\sigma_B^2 = 22$. To test whether the average distance differs between the two batteries, $n_A = 9$ vehicles using battery (A) and $n_B = 11$ vehicles using battery (B) were randomly selected, giving distances X_{Ai} ($i = 1, \dots, 9$) and X_{Bj} ($j = 1, \dots, 11$). Conduct a hypothesis test on the difference of two means at significance level $\alpha = 0.05$. [55 points]

(1) State the null and alternative hypotheses.

(2) Find the null distribution of the test statistic

$$Z = \frac{\bar{X}_A - \bar{X}_B - 0}{\sqrt{\frac{\sigma_A^2}{n_A} + \frac{\sigma_B^2}{n_B}}},$$

where $\bar{X}_A$ is the sample mean for the new battery and $\bar{X}_B$ is the sample mean for the existing battery.

(3) Find the rejection region at significance level $\alpha = 0.05$.

(4) The following summary statistics were obtained. Find the observed value of the test statistic in (2).

$$\bar{x}_A = \frac{1}{9} \sum_{i=1}^9 x_{Ai} = 455, \quad \bar{x}_B = \frac{1}{11} \sum_{j=1}^{11} x_{Bj} = 449.5455$$

(5) Using the results in (3) and (4), complete the hypothesis test. State the conclusion in the context of the data.

2. We wish to investigate whether there is a positive linear relationship between the summer daily maximum temperature X and a café's delivery sales Y . To do so, $n = 31$ days were randomly sampled, and for each day the maximum temperature and the café's delivery sales were recorded, giving the results below. Assume that (X, Y) follows a bivariate normal distribution. [45 points]

i	1	2	3	4	5	6	7	...	30	31
Max. temperature (x_i, °C)	33.2	32.5	34.2	36.2	33.7	33.5	33.4	...	32.9	33
Delivery sales (y_i, 10,000 KRW)	30	25	40	55	20	27	22	...	30	40

- (1) State the null and alternative hypotheses.

- (2) Find the null distribution of the test statistic

$$T = \frac{R\sqrt{n-2}}{\sqrt{1-R^2}},$$

where R is the sample correlation coefficient.

- (3) Find the rejection region at significance level $\alpha = 0.05$.

- (4) The R code and its partial output for this problem are shown below. Use them to find the p -value and complete the hypothesis test at significance level $\alpha = 0.05$. State the conclusion in the context of the data.

```
> temp <- c(33.2, 32.5, 34.2, 36.2, 33.7, 33.5, 33.4, ..., 32.9, 33)
> sales <- c(30, 25, 40, 55, 20, 27, 22, ..., 30, 40)
> cor.test(temp, sales, alternative = 'greater')
```

Pearson's product-moment correlation

```
data: temp and sales
t = 4.0776, df = _____, p-value = _____
alternative hypothesis: true correlation is greater than 0
95 percent confidence interval:
 0.3696727 1.0000000
sample estimates:
      cor
0.603664
```