Introduction to Statistics - Quiz #4(60 minutes)

June 4, 2026 (Thursday)

Section(교반): _____ Cadet Number(교번): ____________ Name(성명): __________________ Score: ____________

- All solutions must include a detailed step-by-step explanation.
- If an answer has more than four decimal places, round to the **fourth decimal place**.
- **Reference table** is provided on the last page of the exam.

1. The driving distance per full charge (in km) of an electric vehicle is known to follow a normal distribution: with a new battery (A), mean μ_A and variance $\sigma_A^2 = 18$; with the existing battery (B), mean μ_B and variance $\sigma_B^2 = 22$. To test whether the average distance differs between the two batteries, $n_A = 9$ vehicles using battery (A) and $n_B = 11$ vehicles using battery (B) were randomly selected, giving distances X_{Ai} $(i = 1, \ldots, 9)$ and X_{Bj} $(j = 1, \ldots, 11)$. Conduct a hypothesis test on the difference of two means at significance level $\alpha = 0.05$. [55 points]

(1) State the null and alternative hypotheses.

Solution:

$$H_0 : \mu_A - \mu_B = 0, \qquad H_A : \mu_A - \mu_B \neq 0$$

(2) Find the null distribution of the test statistic

$$Z = \frac{\bar{X}_A - \bar{X}_B - 0}{\sqrt{\dfrac{\sigma_A^2}{n_A} + \dfrac{\sigma_B^2}{n_B}}},$$

where $\bar{X}_A$ is the sample mean for the new battery and $\bar{X}_B$ is the sample mean for the existing battery.

Solution: Both population variances are known and $\bar{X}_A - \bar{X}_B \sim N(\mu_A - \mu_B, \sigma_A^2/n_A + \sigma_B^2/n_B)$, so under H_0,

$$Z = \frac{\bar{X}_A - \bar{X}_B - 0}{\sqrt{\frac{\sigma_A^2}{n_A} + \frac{\sigma_B^2}{n_B}}} = \frac{\bar{X}_A - \bar{X}_B}{\sqrt{4}} \sim N(0,\, 1).$$

(3) Find the rejection region at significance level $\alpha = 0.05$.

Solution: This is a two-sided test with $\alpha = 0.05$, so the rejection region is

$$|z| > z_{0.025} = 1.96.$$

(4) The following summary statistics were obtained. Find the observed value of the test statistic in (2).

$$\bar{x}_A = \frac{1}{9}\sum_{i=1}^{9} x_{Ai} = 455, \qquad \bar{x}_B = \frac{1}{11}\sum_{j=1}^{11} x_{Bj} = 449.5455$$

Solution: Since $\bar{x}_A = 455$ and $\bar{x}_B = 449.5455$,

$$z = \frac{455 - 449.5455}{\sqrt{\frac{18}{9} + \frac{22}{11}}} = 2.7273.$$

(5) Using the results in (3) and (4), complete the hypothesis test. State the conclusion in the context of the data.

Solution: The observed statistic $z = 2.7273$ lies in the rejection region $|z| > 1.96$, so we reject H_0.

Conclusion: At significance level $\alpha = 0.05$, there is sufficient evidence to conclude that the average distance of the new battery differs from that of the existing battery.

2. We wish to investigate whether there is a positive linear relationship between the summer daily maximum temperature X and a café's delivery sales Y. To do so, $n = 31$ days were randomly sampled, and for each day the maximum temperature and the café's delivery sales were recorded, giving the results below. Assume that (X, Y) follows a bivariate normal distribution. [45 points]

i	1	2	3	4	5	6	7	$\cdots$	30	31
Max. temperature (x_i, °C)	33.2	32.5	34.2	36.2	33.7	33.5	33.4	$\cdots$	32.9	33
Delivery sales (y_i, 10,000 KRW)	30	25	40	55	20	27	22	$\cdots$	30	40

(1) State the null and alternative hypotheses.

Solution:

$$H_0 : \rho = 0, \qquad H_A : \rho > 0$$

(2) Find the null distribution of the test statistic

$$T = \frac{R\sqrt{n-2}}{\sqrt{1-R^2}},$$

where R is the sample correlation coefficient.

Solution: With $n = 31$, under H_0,

$$T = \frac{R\sqrt{n-2}}{\sqrt{1-R^2}} = \frac{R\sqrt{29}}{\sqrt{1-R^2}} \sim t(29).$$

(3) Find the rejection region at significance level $\alpha = 0.05$.

Solution: This is a right-sided test, so the rejection region is $t > t_\alpha(n-2)$. With $\alpha = 0.05$ and $n = 31$,

$$t > t_{0.05}(29) = 1.6991.$$

(4) The R code and its partial output for this problem are shown below. Use them to find the p-value and complete the hypothesis test at significance level $\alpha = 0.05$. State the conclusion in the context of the data.

```
> temp <- c(33.2, 32.5, 34.2, 36.2, 33.7, 33.5, 33.4, ..., 32.9, 33)
> sales <- c(30, 25, 40, 55, 20, 27, 22, ..., 30, 40)
> cor.test(temp, sales, alternative = 'greater')
```

```
        Pearson's product-moment correlation

data:  temp and sales
t = 4.0776, df = ________ , p-value = ___________
alternative hypothesis: true correlation is greater than 0
95 percent confidence interval:
 0.3696727 1.0000000
sample estimates:
     cor
0.603664
```

Solution: The observed test statistic is $t = 4.0776$ with $df = n - 2 = 29$. For a right-sided test,

$$p\text{-value} = P(T > 4.0776) = 0.0002.$$

Since the p-value is smaller than $\alpha = 0.05$, we reject H_0.

Conclusion: At significance level $\alpha = 0.05$, there is sufficient evidence of a positive correlation between the summer daily maximum temperature and the café's delivery sales.