

Introduction to Statistics - Homework #2

Exercise 3.34

An airline charges the following baggage fees: \$25 for the first bag and \$35 for the second. Suppose 54% of passengers have no checked luggage, 34% have one piece of checked luggage, and 12% have two pieces. We assume a negligible portion of people check more than two bags.

(1) Let random variable X represent the baggage fee per passenger. Find $E(X)$ and $SD(X)$.

(2) About how much revenue should the airline expect for a flight of 120 passengers? Given that each passenger's baggage fee is independent, what is the standard deviation of the revenue for a flight of 120 passengers?

Exercise 3.47

Suppose we have n independent observations $X_1, X_2, \dots, X_n$ from a distribution with mean μ and standard deviation σ . What is the expectation and variance of the sample mean of these n values:

$$\bar{X} = \frac{X_1 + X_2 + \dots + X_n}{n}?$$

Extra problem 1

Suppose that X and Y have a joint probability density function

$$f(x, y) = \begin{cases} \frac{3}{2}y^2, & 0 \leq x \leq 2, 0 \leq y \leq 1 \\ 0, & \text{otherwise.} \end{cases}$$

(1) Find the marginal pdf of X and Y , $f_X(x)$ and $f_Y(y)$.

(2) Determine if X and Y are independent.

(3) Find $E(Y)$ and $\text{Var}(Y)$.

Extra problem 2

Suppose that X and Y have a joint probability mass function (pmf)

$$f(x, y) = \begin{cases} \frac{1}{12}(x + y), & x \in \{0, 2\}, y \in \{0, 1, 2\}, \\ 0, & \text{otherwise.} \end{cases}$$

(1) Find the marginal pmf of X , $f_X(x)$.

(2) Find the marginal pmf of Y , $f_Y(y)$.

(3) Find $E(X)$ and $E(Y)$.

(4) Find $E(XY)$ and $\text{Cov}(X, Y)$.

Extra problem 3

Suppose that X and Y have a joint probability density function

$$f(x, y) = \begin{cases} e^{-x-y}, & x > 0, y > 0, \\ 0, & \text{otherwise.} \end{cases}$$

(1) Find the marginal densities $f_X(x)$ and $f_Y(y)$.

(2) Find $E(X)$ and $E(Y)$.

(3) Find $E(XY)$ and $\text{Cov}(X, Y)$.

(4) Find $P(X < Y)$.